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Exam III

Ayman Badawi

QUESTION 1. Let $n = 25(7^2)(2^3)$ and let D be the set of all divisors of n.

(i) Find $|D|$.

(ii) Find $\sigma(n)$, i.e., the sum of all divisors of n.

(iii) Let $F = \{d \in D$, such that, $28 \mid d\}$. Find $|F|$. Find $\sum_{f \in F} f$.

QUESTION 2. (Show the work) Assume that a, b, c are positive integers such that $a \mid bc$. Assume that $gcd(a, b) = 1$. Prove that $a \mid c$.

QUESTION 4. Prove the converse of Q5.

QUESTION 5. Show the work. Use the algorithm as in Q5 and Q6.

(i) Is 5,656 divisible by 101?

(ii) Is 12,423 divisible by 101?

(iii) Is 54,134 divisible by 101?

QUESTION 6. Find the largest positive integer n such that $(n + 21) \mid (3n^4 + 5n + 10)$.

(b) Find $4 - R(U(26))$.

(c) Solve for x over U(26), $x^4 = 3$.

(d) Find all integers over Z, say x, such that $gcd(x, 26) = 1$ and $x^4\ (mod\ 26) = 3$.

QUESTION 8. Prove that $(p-1)!\ \ (mod\ \ p) = p-1$ for every odd prime positive integer.

QUESTION 9. (show the work) Find all positive prime integers, say p, such that $p \mid (389^p + 1)$.

QUESTION 10. Let $m > 1$ be an integer and $f(n) = n^m + a_{m-1}n^{m-1} + ... + a_1 n + a_0$, where all the $a_i's$ are integers and $n \in Z$. Given $f(b_1) = f(b_2) = 22$ for some distinct $b_1, b_2 \in Z$. Prove that $f(k) \neq 23$ for every $k \in Z$.

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